50 Years of Seminar for Analysis and Foundation of Mathematics
led by Academician Bogoljub Stanković

International Conference
Contemporary Problems of Mechanics and Applied Mathematics

Novi Sad, September 3-6, 2012

MechAM2012

Book of Abstracts

Department of Mechanics
University of Novi Sad
2012

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Foreword

The Seminar on mathematical analysis at the Novi Sad University was founded by Professor Bogoljub Stanković in1962. More than hundred mathematicians of the Novi Sad University started their scientific work through this seminar. A large number of distinguished visitors participated at the seminar as well. From the very beginning a vast scope of mathematical topics was covered, starting from integral transforms, integral equations and theory of generalized functions, foundations of mathematics, differential equations and asymptotic behavior of solutions through numerical solutions, fixed point theory and applications.

At the present time the Seminar resembles the enlarged mathematical interest of the researchers participating at the three research projects. Functional analysis topics include Generalized functions as framework for singular ODE and PDE, Microlocal analysis and ΨDO, Integral transforms and asymptotics, Time-frequency analysis (research led by Academician Stevan Pilipović), Mathematical logic and general topology (research led by Professor Miloš Kurilić) and Differential equations with fractional derivatives and their applications (research led by Academician Teodor M. Atanacković).

The Department of Mathematics and Informatics, Department of Mechanics and the Center for Mathematical Research of Nonlinear Phenomena at Novi Sad University, and the Scientific and Organizing Committees are pleased to welcome you to the celebration of 50 years of Seminar for analysis and foundation of mathematics, successfully led by Academician Bogoljub Stanković.

The main programme consists of three conferences:

Topics in PDE, Microlocal and Time-frequency Analysis, September 3-8, 2012

Contemporary Problems of Mechanics and Applied Mathematics, September 3-6, 2012

Mathematical Logic and General Topology, September 5-8, 2012

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Serbian Ministry of Education and Science

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Schedules

	Mon 3 Sep	Tue 4 Sep	Wed 5 Sep
09.30-10.10	Opening + Pilipović	Mainardi	Rodino
10.15-10.55	Oberguggenberger	Spasić	Ruggeri
11.00-11.25	**Coffee**	**break**	
11.25-12.05	Gorenflo	Makris	Jarić
12.10-12.50	Seyranian	Stojanović	Lazarević
13.00-15.20	**Lunch**	**break**	
15.20-15.40	Zorica	Oparnica	Stevanović
15.45-16.05	Tsankov	Janev	Pavić
16.10-16.30	Bazhlekova	Dolićanin	Madjarević
16.35-17.00	**Coffee**	**break**	
17.00-17.20	Takači Dj.	Janevski	Marić
17.25-17.45	Takači A.	Glavardanov	Hadžić
17.50-18.10		Vesković	Hadžić

Monday, September 3

9.30-10.10	Opening + Stevan Pilipović	*Classes of generalized functions with finite type regularities*
10.15-10.55	Michael Oberguggenberger	*Detection of singularities in hyperbolic PDEs via asymptotic properties of generalized solutions*
11.00-11.25	coffee break	
11.25-12.05	Rudolf Gorenflo	*On subordination in time-fractional stochastic processes*
12.10-12.50	Alexander Seyranian	*Paradox of Nicolai and similar effects in non-conservative stability problems*
13.00-15.20	lunch break	
15.20-15.40	Dušan Zorica	*Forced oscillations of a body attached to a light fractional viscoelastic rod*
15.45-16.05	Yulian Tsankov	*Operational calculi for multivariate evolution boundary value problems*
16.10-16.30	Emilia Bazhlekova	*Fractional Differential Equations: Abstract Theory and Some Nonlocal Boundary-Value Problems*
16.35-17.00	coffee break	
17.00-17.20	Djurdjica Takači	*On the approximate solutions of the fuzzy fractional differential equation*
17.25-17.45	Arpad Takači	*On the approximate solutions of the fractional differential equation*

Tuesday, September 4

9.30-10.10	Francesco Mainardi	*On completely monotone and Bernstein functions in relaxation and creep processes*
10.15-10.55	Dragan Spasić	*Engineering Problems with both Nonsmooth Multifunctions and Noninteger Order Derivatives*
11.00-11.25	coffee break	
11.25-12.05	Nicos Makris	*From Hooke's "Hanging Chain" and Milankovitch's "Druckkurven" to a variational formulation: The adventure of the thrust-line of masonry arches*
12.10-12.50	Mirjana Stojanović	*Fractional Helmholtz equation with singularities*
13.00-15.20	lunch break	
15.20-15.40	Ljubica Oparnica	*Euler-Bernoulli beam on the viscoelastic foundation*

15.45-16.05	Marko Janev	*Image denoising by a direct variational minimization*
16.10-16.30	Diana Dolićanin	*An equation with distributed order symmetrized fractional derivative*
16.35-17.00		coffee break
17.00-17.20	Goran Janevski	*A Numerical Method for Estimating the Vibrations of an Viscoelastic Beam*
17.25-17.45	Valentin Glavardanov	*Buckling and postbuckling analysis of nanotube*
17.50-18.10	Miroslav Vesković	*Asymptotic solutions of quasilinear equations and the problem of instability of equlibria of mechanical systems*
19.30		Conference dinner

Wednesday, September 5

9.30-10.10	Luigi Rodino	*Weyl asymptotics and Dirichlet divisors*
10.15-10.55	Tommaso Ruggeri	*Extended thermodynamics of real gases*
11.00-11.25		coffee break
11.25-12.05	Jovo Jarić	*Anisotropic elasticity damage tensor in continuum damage mechanics*
12.10-12.50	Mihailo Lazarević	*Some applications of fractional calculus on control problems in robotics and system stability*
13.00-15.20		lunch break
15.20-15.40	Nevena Stevanović	*Microbearing gas flow modeling by fractional derivative for entire Knudsen number range*
15.45-16.05	Milana Pavić	*Diffusion asymptotics of a kinetic model for gaseous mixtures*
16.10-16.30	Damir Madjarević	*Shock structure and temperature overshoot in multi-temperature model of binary mixture*
16.35-17.00		coffee break
17.00-17.20	Vojislav Marić	*An asymptotic analysis of solutions to equations of Thomas-Fermi type*
17.25-18.05	Olga Hadžić	*50 years of Seminar for analysis and foundations of mathematics*

Thursday, September 6

20.00	Joint conference dinner

Abstracts of Talks

Classes of generalized functions with finite type regularities

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We analyze regularity properties of elements of generalized function algebras parallel to the corresponding theory within distribution spaces. In this sense we considered subspaces or subalgebras which correspond to Sobolev, Zygmund and Hölder spaces. Moreover, we investigate regularity propetoes of Schwartz distributons within the Besov spaces of functions and distributions. In this case, instead of growth order of the form "$O(\varepsilon^a)$", we consider weighted integrals from zero to one with respect to the measure $d\varepsilon/\varepsilon$.

This talk is based on a joint work of Pilipović, Scarpalezos and Vindas, as well as of Pilipović and Vindas.

Detection of singularities in hyperbolic PDEs via asymptotic properties of generalized solutions

Michael Oberguggenberger

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The talk addresses propagation of singularities in linear hyperbolic systems with non-smooth coefficients. The existence of distributional solutions requires a minimal degree of regularity of the coefficients. In case of more singular coefficients, e.g., discontinuous coefficients, solutions may still be constructed in algebras of generalized functions, like the Colombeau algebras.

These generalized functions are represented by families of smooth functions depending on a parameter ε. Classical notions for locating the singularities, such as the wave front set, have a generalization and refinement in the setting of Colombeau algebras in terms of asymptotic estimates with respect to ε.

The talk addresses recently established possibilities of tracing the singularities issuing from the initial data across singularities of the coefficients. Methods of proof involve the generalized wave front set, commutators of vector fields, and Fourier integral operators.

The talk is based on joint work with Hideo Deguchi, Claudia Garetto and Günther Hörmann.

On subordination in time-fractional stochastic processes

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This lecture consists of two parts: A and B. In part A we present two pathways to subordination in space-time fractional diffusion. In Part B we show that the time-fractional Poisson process can be obtained from the classical Poisson process by a time change via the inverse stable subordinator.

Part A_1: The uncoupled spatially one-dimensional Continuous Time Random Walk (CTRW) under power law regime is split into three distinct random walks: a walk (rw1) along the line of natural time, happening in operational time, a walk (rw2) along the line of space, happening in operational time, a walk (rw3) (the walk (rw1) inverted) along the line of operational time, happening in natural time. Via the general integral equation of CTRW and appropriate rescaling, the transition to the diffusion limit is carried out separately for each of these three random walks. Combining the limits of (rw1) and (rw2) we get the method of parameteric subordination for generating particle paths, whereas combination of (rw2) and (rw3) yields the subordination integral formula for the sojourn probability density in space-time fractional difusion.

Part A_2: Via Fourier-Laplace manipulations of the relevant fractional differential equation we obtain the subordination integral formula that teaches us how a particle path can be constructed by first generating the operational time from the physical time and then generating in operational time the spatial path. By inverting the generation of the operational time from the physical time we arrive at the method of parametric subordination.

Part B: We generate the fractional Poisson process by subordinating the standard Poisson process to the inverse stable subordinator. Our´analysis is based on Laplace-Laplace transform of the probability densities. First we give an outline of basic renewal theory, then of the essentials of the classical Poisson process and its fractional generalization via replacement of the exponential waiting time density by one of Mittag-Lefler type. Turning our attention to the probability of the counting number of the fractional Poisson process assuming a given value we find in the transform domain a formula analogous to the Cox-Weiss formula in the theory of continuous time random walk. This formula contains for the jump densities (all increments being positive, in fact equal to 1) the Laplace transform instead of the customary Fourier transform. By manipulating this formula we arrive, after inversion of the transforms, to a subordination integral involving the inverse stable subordinator. Stochastic interpretation of this integral leads to the result that the fractional Poisson process can be obtained from the classical Poisson process via time change to the inverse stable subordinator.

References

[1] L. Beghin and E. Orsingher: Fractional Poisson processes and related random motions, Electronic Journ. Prob. No 61 (2009), 1790–1826.

[2] R. Gorenflo, F. Mainardi and A. Vivoli, Continuous time random walk and parametric subordination in fractional diffusion. Chaos, Solitons and Fractals, **34** (2007), 87–103. [E-print: http://arxiv.org/abs/cond-mat/0701126].

[3] R. Gorenflo and F. Mainardi: Subordination pathways to fractional diffusion. Eur. Phys. J. Special Topics 193 (2011), 119-132.

[4] R. Gorenflo and F. Mainardi: Parametric subordination in fractional diffusion pocesses. In: S.C. Lim , J. Klafter and R. Metzler (Editors), Fractional Dynamics, Chapter 10, pp. 228-261, World Scientific, Singapore, 2011.

[5] F. Mainardi, R. Gorenflo and E. Scalas: A fractional generalization of the Poisson processes. Vietnam Journal of Mathematics 32 SI (2004), 53-64. [E-print http://arxiv.org/abs/math/0701454]

[6] F. Mainardi, Yu. Luchko and G. Pagnini: The fundamental solution of the space-time fractional diffusion equation. Fractional Calculus and Applied Analysis **4** (2001), 153-192.

[7] M.M. Meerschaert, E. Nane and P. Vellaisamy: The fractional Poisson process and the inverse stable subordinator, Electronic Journ. Prob. 16 (2011), 1600-1620.

Paradox of Nicolai and similar effects in non-conservative stability problems

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We present a general approach to the paradox of Nicolai and related effects analyzed as a singularity of the stability boundary. We study potential systems with arbitrary degrees of freedom and two coincident eigenfrequencies disturbed by small non-conservative positional and damping forces. The instability region is obtained in the form of a cone having a finite discontinuous increase in the general case when arbitrarily small damping is introduced. This is a new destabilization phenomenon, which is similar to the effect of the discontinuous increase of the combination resonance region due to addition of infinitesimal damping. Then we consider the paradox of Nicolai: the instability of a uniform axisymmetric elastic column loaded by axial force and a tangential torque. It is shown that the paradox of Nicolai is related to the conical singularity of the stability boundary which transforms to a hyperboloid with the addition of small dissipation.

Forced oscillations of a body attached to a light fractional viscoelastic rod

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We study forced oscillations of a body attached to a free end of a viscoelastic rod (the other end of the rod is fixed). We presented the case when the mass of the rod is negligible in comparison with the mass of the body. Constitutive equation for the rod is assumed in a general form. The existence of the solution for displacement and stress is proved and several numerical examples are presented.

Operational calculi for multivariate evolution boundary value problems

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Direct Mikusiński type operational calculi for several real variables are proposed by various authors. As a rule they are applicable only to Cauchy problems for linear partial differential equation with constant coefficients. As it concerns mixed problems, i.e. problems, containing both boundary and initial conditions these operational calculi are unpractical.

Here, using non-classical convolutions (see [1]), we propose a direct operational calculus approach to nonlocal boundary value problems for a large class of evolution equations with several space variables. To this end we introduce multidimensional convolution algebra and the ring of multipliers fractions of this algebra. Our starting point is the class of linear nonlocal boundary value problems for PDEs of the form:

$$P(\partial_t)u + \sum_{j=1}^{n} Q_j(\partial_{x_j}^2)u = F(x_1,\ldots,x_n,t), \quad 0<t,\ 0<x_j<a_j, \quad j=1,\ldots,n,$$

where P and Q_j, $j=1,\ldots,n$ are polynomials of one variable, with $\deg P \geq 1$ and $\deg Q_j \geq 1$, with the "initial" conditions

$$\chi_\tau\left\{\partial_t^k u(x_1,\ldots,x_n,\tau)\right\} = f_k(x_1,\ldots,x_n), \quad k=0,1,\ldots,\deg P-1,$$

where χ is a non-zero linear functional on $C[0,\infty)$, and boundary value conditions

$$\begin{gathered}\partial_{x_j}^{2m_j}u(x_1,\ldots,x_{j-1},0,x_{j+1},\ldots,x_n,t) = g_{m_j}(x_1,\ldots,x_{j-1},x_{j+1},\ldots,x_n,t)\\ \Phi_{j,\xi}\left\{\partial_{x_j}^{2m_j}u(x_1,\ldots,x_{j-1},\xi,x_{j+1},\ldots,x_n,t)\right\} = h_{m_j}(x_1,\ldots,x_{j-1},x_{j+1},\ldots,x_n,t)\\ j=1,\ldots,n, \quad m_j=0,1,\ldots,\deg Q_j-1\end{gathered}$$

where Φ_j, $j=1,\ldots,n$ are supposed be non-zero linear functionals on $C^1[0,a_j]$.

The operational calculi developed here allow to obtained explicit solutions of series of local and nonlocal evolution boundary value problems.

References

[1] Dimovski, I.H. Convolutional Calculus. Kluwer, Dordrecht. 1990.

Fractional Differential Equations: Abstract Theory and Some Nonlocal Boundary-Value Problems

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During the last few decades a considerable interest has been devoted to the application of fractional calculus modeling to different fields of science e.g. classical and quantum mechanics, rheology, nuclear physics, biology, geomorphology etc. This stimulated the development of the mathematical theory of partial differential equations of fractional order. The existence of a unique solution and its regularity as well as methods of obtaining explicit or approximate solutions are extensively studied.

We consider first the abstract differential equation of fractional order

$$D_t^\alpha u(t) = Au(t) + f(t), \quad t > 0,$$

where D_t^α denotes the Caputo fractional derivative of order $\alpha \in (0, 2)$, A is a closed linear operator densely defined in a Banach space X, $f(t)$ is a given vector-valued function and appropriate initial conditions for $u(t)$ are prescribed. Results on the unique solvability of the problem and some properties of the solution such as analyticity, regularity, perturbation, subordination principle, are presented. It appears that many facts from the theories of C_0-semigroups of operators ($\alpha = 1$) and cosine operator functions ($\alpha = 2$) have natural analogues for the solution operators in the case of noninteger α. However, in some aspects the fractional problems show remarkably different features.

Regularity estimates for the solution in various functional spaces are obtained. Such estimates are essential in the computational stability analysis of numerical solutions, in inverse problems as well as in the transition from linear to quasilinear problems.

Some particular initial-boundary-value problems for the time-fractional diffusion-wave equation are also considered, including problems with nonlocal boundary conditions. Applying the operational calculus approach proposed by Dimovski, we find explicit Duhamel-type representation of the solution. This representation is used for numerical computation and visualization of the solution.

On the approximate solutions of the fuzzy fractional differential equation

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Fuzzy fractional integral and derivative introduced by Agarwal are presented.In that sense the class of fuzzy fractional differential equations are consider and the approximate solutions are constructed and analyzed.

On the approximate solutions of the fractional differential equation

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The time-fractional integro-differential equation of the form

$$\frac{\partial^2 u(x,t)}{\partial t^2} + a\int_0^t k(t-\tau)\frac{\partial^2 u(x,\tau)}{\partial \tau^2}d\tau = b\int_0^t k(t-\tau)\frac{\partial^3 u(x,\tau)}{\partial x^2 \partial \tau}d\tau + \frac{\partial^2 u(x,t)}{\partial x^2},$$

for $t > 0,\ x \in (0,1)$, for $k(t) = \frac{t^{-\alpha}}{\Gamma(1-\alpha)},\quad 0 < \alpha < 1$, with the following conditions:

$$u(0,x) = u_0(x),\quad u_t(0,x) = u_1(x) \qquad u(t,0) = u(t,1) = 0, \qquad t > 0$$

is analyzed in the frames of the Mikusiński calculus. We present a method for obtaining the exact and the approximate solution.

On completely monotone and Bernstein functions in relaxation and creep processes

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In view of the electro-mechanical analogy, linear viscoelastic and dielectric materials exhibit similar features as far as the time dependent response functions are concerned. As a common characteristic, the relaxation functions are completely monotonic (CM) whereas the creep functions are of Bernstein type. This means that these response functions are represented by discrete or continuous distributions of elementary (i.e. exponential) relaxation and creep processes via spectra of relaxation and retardation times with physical relevance. In this talk we will discuss some interesting examples of relaxation and creep processes occurring in viscoelastic or dielectric materials, which are described by special CM and Bernstein functions, which turn out to be of Mittag-Leffler, stretched exponential, logarithmic and power law types.

References

[1] E. Capelas de Oliveira, F. Mainardi and J. Vaz Jr: Models based on Mittag-Leffler functions for anomalous relaxation in dielectrics, Eur. Phys. J., Special Topics, Vol. 193, pp. 161-171 (2011). E-print http://arxiv.org/abs/1106.1761

[2] R. Gorenflo and F. Mainardi: Fractional relaxation of distributed order, in M. Novak (Editor): Complexus Mundi: Emergent Patterns in Nature, World Scientific, Singapore (2006), pp. 33-42.

[3] A. Hanyga and M. Seredynska: On a mathematical framework for the constitutive equations of anisotropic dielectric relaxation. J. Stat. Phys, Vol 131, pp. 269-303 (2008).

[4] F. Mainardi: Fractional Calculus and Waves in Linear Viscoelasticity, Imperial College Press, London (2010), pp. 340.

[5] F. Mainardi, A. Mura, R. Gorenflo and M. Stojanovic: The two forms of fractional relaxation of distributed order. Jour. Vibr. Control., Vol 13 No 9/10, pp. 1249-1268 (2007). E-print http://arxiv.org/abs/cond-mat/0701131

[6] F. Mainardi and G. Spada: On the viscoelastic characterization of the Jeffreys-Lomnitz law of creep. Rheol. Acta, published on line 16/05/2012, DOI 10.1007/s00397-012-0634-x, E-print: http://arxiv.org/abs/1112.5543.

[7] F. Mainardi and G. Spada: Becker and Lomnitz rheological models: a comparison, AIP (Amer. Inst. of Physics) Proceedings of Int. Conference "Times of Polymers and Composites", Ischia, Italy 10-14 June 2012.

[8] K.S. Miller and S.G. Samko: Completely monotonic functions, Integr. Transf. and Spec. Funct., Vol pp. 389-402 (2001).

[9] R.L. Schilling, R. Song and Z. Vondracek: Bernstein Functions: Theory and Applications, De Gruyter, Berlin, 2010.

On Engineering Problems with both Nonsmooth Multifunctions and Noninteger Order Derivatives

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Because of today's concern for liability, engineering innovations must be exhaustively tested and analytically proven on models generated by fundamental physical and geometrical principles which are complemented with an appropriate set of constitutive equations. In order to formulate a well-posed problem and to allow accurate predictions of its behavior, certain hypothesis should be made and a rheological description of the system component that contains enough information on its physical properties should be chosen. Among the variety of all possible choices that can be used in mechanical problems dealing with impacts and oscillatory motions, we suggest the constitutive model of the viscoelastic body with fractional derivatives of stress and strain, restrictions on the coefficients that follow from Clausius-Duhem inequality, and discontinuous inequality constraint conditions imposed by the Coulomb friction model. The principal advantages of the model are twofold. First, it takes an energy dissipation *ab initio*, and secondly, it can be used for rheological description of both new high performance materials, such as elastomers/polymers, as well as different biological tissues. Besides, since it contains both nonsmooth multifuctions and nonlocal operators, it possess an essential mathematical interest too. Thus, we give three examples belonging to parallel studies of fractional differential equations to the well known theory of ordinary differential equations and show possible connections between fractional calculus and nonsmooth dynamics from both theoretical and numerical point of view.

First, we study dynamics of a block, sliding on a dry surface and impacting against another block through a standard fractional viscoelastic body, that we model as a straight rod of negligible mass. Due to the presence of dry friction and the proposed fractional model the problem belongs to the class of set-valued fractional differential equations (or multivalued differential equations of arbitrary real order) leading to the Cauchy problem for two coupled integro-differential inclusions, for which the existence result ensuring the contractible solution set exists. By use of the slack variable algorithm the problem was solved numerically. It was shown that both separation and capture behavior patterns of the blocks after impact are predictable. Actually, there are ten different scripts for different values of the system parameters. On the other side, using the Atanackovic-Stankovic expansion formula for fractional derivatives, the same problem can be transformed into the system of differential equations of integer order of the Fillipov type. Special features of latter approach are that the existence of the solution for fractional differential inclusion can be proved by the classical result and numerical solutions are obtained by standard numerical procedures.

Next, we deal with a seismic base isolation problem. Namely, we study dynamics of a column-like structure consisting of two blocks positioned one on another with passive dumping elements between them. We assume that there is a sliding friction between the blocks, which was modeled by a set valued function, as well as the connection between the blocks that contains a standard fractional viscoleastic body. We discuss the simplified earthquake models, i.e. the exponentially decreasing sinusoidal function as well as the Rickert type of ground motion. The dynamics of the problem is given in form of a system of set-valued fractional differential equations. By use of the Laplace transform method we show that there is a periodical solution of the obtained equivalent Cauchy problem for coupled integro-differential inclusions that corresponds to the slip-stick phase of the motion after the earthquake. The suggested numerical procedure for solving the problem was based on the Grünwald-Letnikov form of the fractional derivative and the slack variable algorithm used for handling discontinuous model motion phases. Results for different loads and the dumper parameters are discussed. Some alternative formulations of the problem as well as numerical procedures for its solutions are also considered.

Finally, we pose an optimal control problem for a sliding-isolated seismic-excited structure with passive fractional damping. Namely, we intend to keep the seismically excited structure near the equilibrium in prescribed time by minimizing the control force as well. As in the previous example we use the fractional Zener model for the passive damping. Referring again to the Atanackovic-Stankovic expansion formula for RL-fractional derivatives in terms of function, its integer-order derivatives and moments, we use the Pontryagin maximum principle to derive necessary conditions for optimality. Noting that the nonlocal effects are taken into account through moments of the functions involved and that each of the moments, requires corresponding adjoint variable, we comment on the order of the obtained equivalent system and numerical solutions of the problem obtained by the Krilov-Chernousko method of successive approximations. Several related optimal control problems including the one in which the motion of the systems ceases in a finite time are discussed at the end.

From Hooke's "Hanging Chain" and Milankovitch's "Druckkurven" to a variational formulation: The adventure of the thrust-line of masonry arches

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More than a century ago the Serbian engineer and astronomer Milutin Milankovitch presented a remarkable formulation for the thrust-line of arches that do not sustain tension, and using polar coordinates he presented for the first time the correct and complete solution for the theoretical minimum thickness, t, of a semicircular arch with radius R. This paper shows that Milankovitch's solution, $t/R = 0.1075$, is not unique and that it depends on the coordinate system used. The adoption of a cartesian coordinate system yields a neighboring thrust-line and a different, slightly higher value for the minimum thickness ($t/R = 0.1095$) than the value computed by Milankovitch. This result has been obtained recently (Makris and Alexakis 2012) with a geometric and a variational formulation.

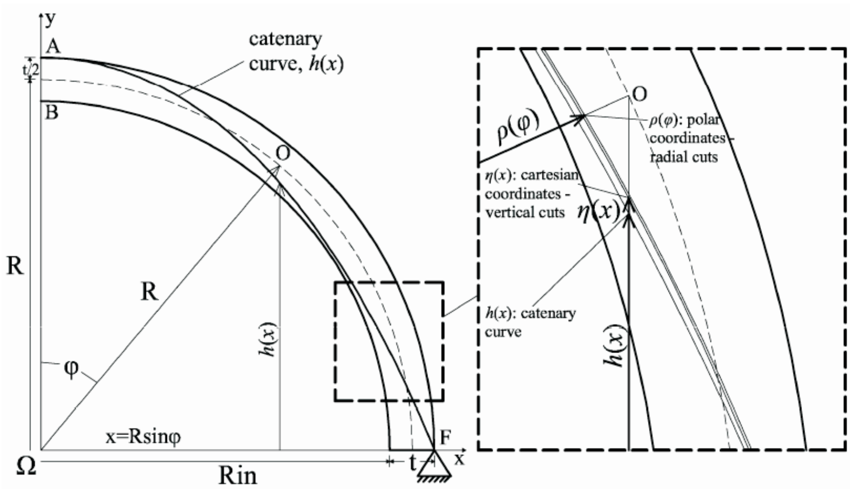

Figure 1. Left: Semicircular arched monolith with $t/R = 0.12$ which accommodates a catenary curve (thin solid line) that passes by the extrados springing point F and is long enough to be tangent at the extrados point A at the crown. Right: The catenary curve, $h(x)$ is different than the two physically admissible minimum thrust-lines $\rho(\varphi)$ given by Milankovitch (1904, 1907) and $\eta(x)$ given by Makris and Alexakis (2012).

The Milankovitch minimum thrust-line derived with a polar coordinate system and our minimum thrust-line derived with a cartesian coordinate system are two distinguishable, physically admissible thrust-lines which do not coincide with Hooke's catenary (1675) that meets the extrados of the arch at the three extreme points. For instance, Figure 1 shows: (a) the radius of curvature $\rho(\varphi)$ of the minimum thrust-line as derived by Milankovitch (1904, 1907) after adopting a polar coordinate system; (b) the abscissa $\eta(x)$ of the neighboring minimum thrust-line derived by Makris and Alexakis (2012) after adopting a cartesian coordinate system; and (c) the abscissa $h(x)$ of Hooke's "hanging chain" that meets the extrados of the arch at the three extreme points.

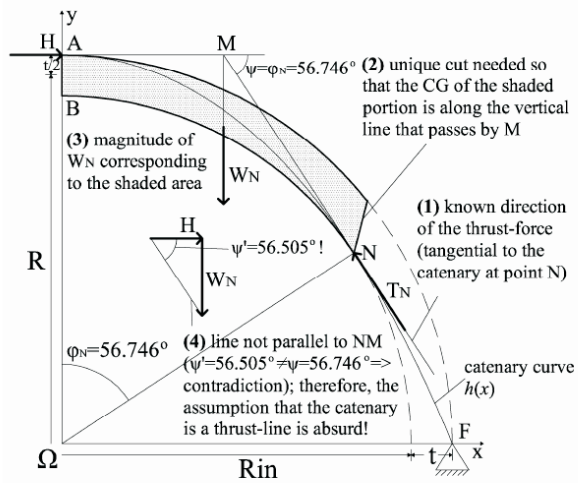

Figure 2. Equilibrium check at $\varphi_N = 56.746°$ shows by reducing to the absurd that the catenary curve (the "hanging chain") which can only just be located within the semicircular arch is not a physically admissible thrust-line.

Most importantly, the paper shows that the catenary (the "hanging chain") is not a physically admissible minimum thrust-line of the semicircular arch although it is a neighboring line to the aforementioned physically admissible thrust-lines. Figure 2 portrays the equilibrium check and the location where the catenary curve touches the intrados ($\varphi_N = 56.746°$) of a semicircular monolith that does not sustain tension and it is shown by reducing to the absurd that the catenary curve (the "hanging chain") which can only just be located within the semicircular arch is not a physically admissible thrust-line.

Polar Coordinate System		**Cartesian Coordinate System**		Minimum thickness of a semicircular arch needed to accommodate a catenary curve
Rupture angle	Minimum thickness ($R=R_{in}$+t/2)	Rupture angle	Minimum thickness ($R=R_{in}$+t/2)	
φ_r=BΩK=54.484°	**t/R=0.10748**	**φ_r=BΩK=54.923°**	**t/R=0.10946**	**t/R=0.11166≈1/9**
• Milankovitch (1904, 1907): Geometric Solution • Ochsendorf (2002): Trial-and-error solution of the "work balance" equation • Makris & Alexakis (2012): Principle of Stationary Potential Energy		• Makris & Alexakis (2012): (a) Geometric Solution (b) Principle of Stationary Potential Energy		• Makris & Alexakis (2012): Geometric Solution The catenary curve is not a physically admissible thrust-line; therefore the point that touches the intrados of the arch (φ_N=56.746°) is not an imminent hinge.

Table 1. Minimum allowable thickness and rupture locations of a semicircular monolith with zero tensile strength.

The minimum thickness of a semicircular arch that is needed to accommodate the catenary curve is $t/R = 0.1117$ – a value that is even higher than the enhanced minimum thickness $t/R = 0.1095$ computed in this paper after adopting a cartesian coordinate system; therefore, it works towards the safety of the arch. Accordingly, Heyman's (1969) solution remains unconservative regardless the stereotomy exercised on the arch – even if one assumes vertical joints (Heyman, 2009).

The results derived in this work for the minimum allowable thickness and the rupture location of a semicircular monolith with zero tensile strength subjected to its own weight is summarized in Table 1 together with the list of past publications which derived the correct results with various approaches.

References

Heyman, J., 1969. The safety of masonry arches. Int. J. mech. Sci. 11, 363-385.

Heyman, J., 2009. La Coupe des Pierres. Proc. 3rd int. cong. on Construction History, Cottbus, 2, pp 807-812.

Hooke, R., 1675. A description of helioscopes, and some other instruments. London.

Ochsendorf, J., 2002. Collapse of masonry structures. PhD thesis, Dept. of Engineering, Univ. of Cambridge, U.K.

Makris, N., and Alexakis, H., 2012. The effect of stereotomy on the shape of the thrust-line and the minimum thickness of semicircular masonry arches. Int. J. Solids Struct. (under review).

Milankovitch, M., 1904. Beitrag zur Theorie der Druckkurven. Dissertation zur Erlangung der Doktorwürde, K.K. technische Hochschule, Vienna.

Milankovitch, M., 1907. Theorie der Druckkurven. Zeitschrift für Mathematik und Physik 55, 1-27.

Fractional Helmholtz equation with singularities

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We prove an existence-uniqueness result for an initial value problem with singularities for nonlinear fractional Helmholtz equation of fractional order α, where $1 < Re(\alpha) \leq 2$. As a framework, we employ Colombeau algebra of generalized functions containing fractional derivatives and operations among them in order to deal with the fractional equations with singularities.

Euler-Bernoulli beam on the viscoelastic foundation

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Recently, in [3], we studied the initial-boundary value problem for an Euler-Bernoulli beam model with discontinuous bending stiffness laying on a viscoelastic foundation and subjected to an axial force and an external load both of Dirac-type (cf. [1] for mechanical background).

The differential equation of the transversal motion reads

$$\frac{\partial^2}{dx^2}\left(A(x)\frac{\partial^2 u}{dx^2}\right)+P(t)\frac{\partial^2 u}{\partial x^2}+R(x)\frac{\partial^2 u}{\partial t^2}+g(x,t)=h(x,t), \qquad x\in[0,1],\ t>0, \tag{1}$$

where

- A denotes the bending stiffness and is given by $A(x) = EI_1 + H(x - x_0)EI_2$. Here, the constant E is the modulus of elasticity, I_1, I_2, $I_1 \neq I_2$, are the moments of inertia that correspond to the two parts of the beam, and H is the Heaviside jump function;
- R denotes the line density (i.e., mass per length) of the material and is of the form $R(x) = R_0 + H(x - x_0)(R_1 - R_2)$;
- P is the axial force, and is assumed to be of the form $P(t) = P_0 + P_1\delta(t-t_1)$ with $P_0, P_1 > 0$;
- g represents the force terms associated with the foundation;
- u denotes the displacement of the beam;
- h is the prescribed external load (e.g. when describing moving load it is of the form $h(x,t) = H_0\delta(x - ct)$, H_0 and c are constants).

Since the beam is connected to the viscoelastic foundation there is a constitutive equation describing relation between the force of foundation and the displacement of the beam. The viscoelastic foundation is of Zener type and described by a fractional differential equation with respect to time:

$$D_t^\alpha u(x,t) + u(x,t) = \theta\, D_t^\alpha g(x,t) + g(x,t), \tag{2}$$

where $0 < \theta < 1$, $0 < \alpha < 1$, and D_t^α denotes the left Riemann-Liouville fractional derivative of order α with respect to t, defined by

$$D_t^\alpha u(t) = \frac{1}{\Gamma(1-\alpha)}\frac{d}{dt}\int_0^t \frac{u(\tau)}{(t-\tau)^\alpha}\,d\tau.$$

System (1)-(2) is supplied with initial conditions

$$u(x,0) = f_1(x), \qquad \partial_t u(x,0) = f_2(x),$$

where f_1 and f_2 are the initial displacement and the initial velocity. If $f_1(x) = f_2(x) = 0$ the only solution to (1)-(2) should be $u \equiv g \equiv 0$. Also, the beam is considered to be fixed at both ends, hence boundary conditions take the form

$$u(0,t) = u(1,t) = 0, \qquad \partial_x u(0,t) = \partial_x u(1,t) = 0.$$

By a change of variables $t \mapsto \tau$ via $t(\tau) = \sqrt{R(x)}\tau$ the problem (1)-(2) is transformed into the standard form

$$\partial_t^2 u + Q(t,x,\partial_x)u + g = h, \tag{3}$$

$$D_t^\alpha u + u = \theta\, D_t^\alpha g + g, \tag{4}$$

$$u|_{t=0} = f_1, \quad \partial_t u|_{t=0} = f_2, \tag{IC}$$

$$u|_{x=0} = u|_{x=1} = 0, \quad \partial_x u|_{x=0} = \partial_x u|_{x=1} = 0, \tag{BC}$$

where Q is a differential operator of the form

$$Qu := \partial_x^2(c(x)\partial_x^2 u) + b(x,t)\partial_x^2 u.$$

The function c in (3) equals A and therefore is of Heaviside type, and the function b is then given by $b(x,t) = P(R(x)t)$ and its regularity properties depend on the assumptions on P and R. Problem (3)-(4) is equivalent to

$$\partial_t^2 u + Q(t,x,\partial_x)u + Lu = h, \tag{5}$$

with L being the (convolution) operator given by ($\mathcal{L}$ denoting the Laplace transform)

$$Lu(x,t) = \mathcal{L}^{-1}\left(\frac{1+s^\alpha}{1+\theta s^\alpha}\right)(t) *_t u(x,t), \tag{6}$$

with the same initial (IC) and boundary (BC) conditions.

Standard functional analytic techniques reach as far as the following: boundedness of b together with sufficient (spatial Sobolev) regularity of the initial values f_1, f_2 ensure existence of a unique solution $u \in L^2((0,T); H_0^2((0,1)))$ to (5) with (IC) and (BC). However, the prominent case $b = p_0 + p_1\delta(t-t_1)$ is clearly not covered by such a result, so in order to allow for these stronger singularities one needs to go beyond distributional solutions.

We have set up and solved Equation (5) subject to the initial and boundary conditions (IC) and (BC) in an appropriate space of Colombeau generalized functions on the domain $X_T := (0,1) \times (0,T)$ (with $T > 0$) as introduced in [2] and applied later on, e.g., in [4]. Therefore b, c, g, h, f_1 and f_2 are generalized functions in following sence: one start with regularizing families $(u_\varepsilon)_{\varepsilon\in(0,1]}$ of smooth functions $u_\varepsilon \in H^\infty(X_T)$ (space of smooth functions on X_T all of whose derivatives belong to L^2). We write $(u_\varepsilon)_\varepsilon$ to mean $(u_\varepsilon)_{\varepsilon\in(0,1]}$. Then one consider the following subalgebras: *Moderate families*, denoted by $\mathcal{E}_{M,H^\infty(X_T)}$, are defined by the property

$$\forall\, \alpha \in \mathbb{N}_0^n, \exists\, p \geq 0 : \|\partial^\alpha u_\varepsilon\|_{L^2(X_T)} = O(\varepsilon^{-p}), \quad \text{ as } \varepsilon \to 0.$$

Null families, denoted by $\mathcal{N}_{H^\infty(X_T)}$, are the families in $\mathcal{E}_{M,H^\infty(X_T)}$ satisfying

$$\forall\, q \geq 0 : \|u_\varepsilon\|_{L^2(X_T)} = O(\varepsilon^q) \quad \text{ as } \varepsilon \to 0.$$

Thus moderateness requires L^2 estimates with at most polynomial divergence as $\varepsilon \to 0$, together with all derivatives, while null families vanish very rapidly as $\varepsilon \to 0$. Null families form a differential ideal in the collection of moderate families and we may define the *Colombeau algebra* as the factor algebra

$$\mathcal{G}_{H^\infty(X_T)} = \mathcal{E}_{M,H^\infty(X_T)}/\mathcal{N}_{H^\infty(X_T)}.$$

So, we show how functional analytic methods for abstract variational problems can be applied in combination with regularization techniques to prove existence and uniqueness of generalized solutions to our initial-boundary problem.

The talk is based on joint work with Güenther Hoermann (Faculty of Mathematics, University of Vienna) and Sanja Konjik (Faculty of Sciences, Department of Mathematics and Informatics, University of Novi Sad).

References

[1] Atanacković, T. M. *Stability Theory of Elastic Rods.* World Scientific, Singapore, 1997.

[2] Biagioni, H. A., Oberguggenberger, M. Generalized solutions to the Korteweg-de Vries and the regularized long-wave equations. *SIAM J. Math. Anal.*, **23**:923–940, 1992.

[3] Hörmann, G., Konjik, S., Oparnica, Lj. Generalized solutions for the Euler-Bernoulli model with Zener viscoelastic foundations and distributional forces, *accepted for publication in Analysis and Applications*, preprint on arXiv:1102.2148 [math.AP].

[4] Hörmann, G., Oparnica, Lj. Generalized solutions for the Euler-Bernoulli model with distributional forces. *J. Math. Anal. Appl.*, **357**(1):142–153, 2009.

Image denoising by a direct variational minimization

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Since the work of Perona and Malik , PDE methods have been used for image processing, especially for denoising and stabilizing edges. They were the first to replace an isotropic diffusion expressed through a linear heat equation with an anisotropic diffusion. Diffusion, in generally, is associated with an energy dissipating process. This process seeks the minima of an energy functional. For example, the well known total variation (TV) minimization model is obtained in the case when the energy functional is equal to the TV norm of the image. Although these methods have been demonstrated to be able to achieve a good trade-off between the noise removal and the edge preservation, the resulting image in the presence of the noise often has a ”blocky” look.

In this work, we present a novel variational, and at the same time patch-based image smoothing method, which combines a mathematically well-posdenes of the variational modeling with the efficiency of a patch-based approach. More-over, the proposed method is based on the direct variational minimization of the appropriate energy functional, which involves fractional gradient. By doing so, we avoid problems of finding the optimal stopping time and the optimal time step. The role of λ is sustained and the actual minimization is conducted till it converges (with respect to the predefined error bound of the particular optimization method). We note that patch-based approach is also convenient to make the proposed direct variational method computationally feasible and applicable on real images. Actually, if working with the whole image, one needs a huge approximation bases1, which is not computationally feasible. According to this, we proceed as follows: The image is divided into relatively small overlap- ping patches, and the energy functional is minimized on each particular patch independently by using a direct variational minimization. As patches should not be to small, in order to capture enough relevant image features, the computational load would be still unacceptable for any real application if one calculates the minimizer in the whole orthonormal basis of the particular patch. There-fore, we approximate the true minimizer by using the Ritz variational method with a specially chosen trial functions. In the sequel we call the set of those functions: the approximation generator. For that purpose, we derive the complementary fractional variational principle (CFVP) for the corresponding energy functional. The CFVP gives us the explicit upper bound for the L_2 norm of the approximation error. Next, we proceeded with spatial discretization of the continuous model, i.e., we make transition from the continuous image to pixels.

Every discrete patch is analyzed in the chosen discrete over-complete dictionary (same for every patch) that has the sparsity property in the class of discrete images of interest. In this work, we use a simple discrete cosine transform (DCT) over-complete dictionary which possess a sparsity property in the class of images. The elements of the actual approximation generator for a particular patch, are chosen to be those with the largest $K << N$ projections $\langle \phi_0, \psi_n \rangle$, where N is size of the orthonormal basis and ϕ_0 is observed noisy image, so that the upper error bound obtained by a spatially discretized CFVP is below the predefined threshold. Thus, the computational load is additionally rapidly reduced, making the method applicable for practical purposes. Moreover, as we conduct the minimization of the target functional on each patch separately, we use different values for the Lagrange multiplier for each patch. The choice is based on the measure of nonsmoothness of the signal present on that particular patch which is obtained by an appropriate pre-processing. Thus, we obtain additional stronger regularization on the uniform and weaker regularization on the oscillatory patches, which significantly improves resulting image quality. It is an additional adaptive feature of the proposed method which is not applicable to anisotropic diffusion PDE modeling. Actually, for that purpose anisotropic diffusion uses only an appropriate edge stopping function (in our case "mini-mal surface"), which is also included in the proposed model. We also note that we use the functional that contains gradient of a fractional order, in order to gain all benefits of fractional approach, in comparison to the classical gradient method or the methods of higher order, as it is previously explained.

An equation with distributed order symmetrized fractional derivative

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In this paper we study equation

$$\frac{d^2}{dt^2}u\left(t\right)+b\int_0^1 {}^{\pm}\mathcal{E}_T^{\alpha}u\left(t\right)\phi(\alpha)d\alpha+F\left(u\left(t\right)\right)=0,\; t\in[0,T],\; T>0$$

where, $\int_0^1 {}^{\pm}\mathcal{E}_T^{\alpha}u\left(t\right)\phi(\alpha)d\alpha$ is the distributed order symmetrized Caputo fractional derivative of u, $\phi(\alpha),\alpha\in(0,1)$, is a positive integrable function (it can also be a compactly supported distribution with the support in $(0,1)$) and $F(u), u\in\mathbb{R}$, is locally Lipschitz continuous function in $\mathbb{R}$. We have researched its solvability, dissipativity and stability.

A Numerical Method for Estimating the Vibrations of an Viscoelastic Beam

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We consider viscoelastic Timoshenko beam which governing partial differential equations are

$$\rho A\partial^2_{TT}W - kGA(1+\bar{\nu}\partial^\alpha_{T^\alpha})(\partial^2_{ZZ}W - \partial_Z\Psi) + F(T)\partial^2_{ZZ}W = 0,$$

$$\rho I_x\partial^2_{TT}\Psi - kGA(1+\bar{\nu}\partial^\alpha_{T^\alpha})(\partial_Z W - \Psi) - EI_x(1+\bar{\nu}\partial^\alpha_{T^\alpha})\partial^2_{ZZ}\Psi = 0,$$

where are: A - area of cross-section; I_x- axial moment of inertia; $\bar{\nu}$ - retardation time; ρ - mass density; T - time; Z - beam coordinate; W - transverse displacement; Ψ - banding slope; k - shear correction factor; G - shear modulus; E - Young modulus; $F(T)$ - time-dependent axial compressive load.

By introducing non-dimensional variables: $W = lw,\ Z = lz,\ k_t = l^2\sqrt{\frac{\rho A}{EI_x}},\ T = k_t t,\ \nu = \frac{\bar{\nu}}{k_t}$, and $f(t) = l^2\frac{F(k_t t)}{EI_x}$, $\kappa = l^2\frac{kGA}{EI_x}$, $r = \frac{I_x}{Al^2}$, we obtain

$$\partial^2_{tt}w - \kappa(1+\nu\partial^\alpha_{t^\alpha})(\partial^2_{zz}w - \partial_z\psi) + f(t)\partial^2_{zz}w = 0,$$

$$\partial^2_{tt}\psi - \frac{\kappa}{r}(1+\nu\partial^\alpha_{t^\alpha})(\partial_z w - \psi) - \frac{1}{r}(1+\nu\partial^\alpha_{t^\alpha})\partial^2_{zz}\psi = 0.$$

Supposing the solutions in the form

$$w(z,t) = \sum_{m=1}^{\infty} X_m(t)\sin(\beta_m z), \quad \psi(z,t) = \sum_{m=1}^{\infty} Y_m(t)\cos(\beta_m z),$$

where $\beta_m = m\pi$, we can reduce to the following system of fractional differential equations

$$(D^\alpha X)(t) = a_1(t)X''(t) + b_1(t)Y''(t) + c_1(t)X(t) + d_1(t)Y(t),$$
$$(D^\alpha Y)(t) = a_2(t)X''(t) + b_2(t)Y''(t) + c_2(t)X(t) + d_2(t)Y(t).$$

Let us remind that the fractional integral of order $\alpha \in \mathbb{R}^+$ and Riemman-Liouville fractional derivative of order $\alpha \in \mathbb{R}^+$ defined by

$$J^\alpha[f(t)] = \frac{1}{\Gamma(\alpha)}\int_0^t (t-u)^{\alpha-1}\ f(u)du, \quad D^\alpha[f(t)] = D^{\lceil\alpha\rceil}J^{\lceil\alpha\rceil-\alpha}[f(t)].$$

where $\lceil\alpha\rceil$ is the nearest integer number greater or equal to α.

The Hadamard finite part integral (HFP-integral) for $g(x)$ is

$$H^\mu(g) = (HFP)\int_0^1 \frac{g(x)}{x^\mu}\ dx = I^\mu_k(g) + D^\mu_k(g)$$
$$= \int_0^1 \frac{g(x) - g_k(x)}{x^\mu}\ dx + \sum_{i=0}^{k}\frac{g^{(i)}(0)}{i!(-\mu+i+1)}.$$

Very effective and useful quadrature formulas were developed in (see [1]). For a fixed $q \in (0,1)$, it was developed

$$\int_0^1 \frac{g(u)}{u^{1+q}}\, du \approx Q_j[g] = \sum_{k=0}^{j} \sigma_{k,j}\, g(k/j) \qquad (j \in \mathbb{N}).$$

Using it we can introduce a formula for numerical differentiation

$$D^\alpha[f(t_j)] \approx \frac{1}{\Gamma(-\alpha) t_j^\alpha} \sum_{i=0}^{j} \sigma_{j-i,j} f_i \qquad (j = 1, 2, \ldots, n).$$

The previous theoretical considerations can be used for determination of dynamical characteristics of the Thimoshenko viscoelastic beam.

Example. Let us consider the case

$$\alpha = 1/2, \quad m = 1, \quad \nu = 0.01, \quad r = 0.025, \quad \kappa = 12, \quad f(t) = \cos t.$$

The results are shown on the Figure 1.

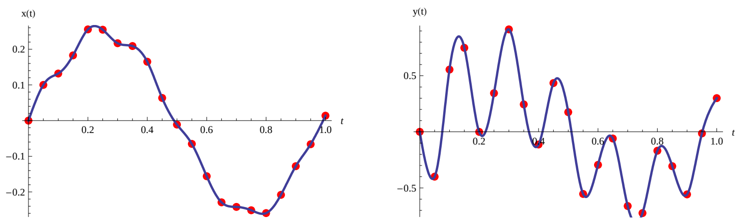

Figure 1: a) $x(t)$ b) $y(t)$

References

[1] Diethelm, K., Ford, N.J., Freed, A.D., Luchko, Yu., (2005), *Algorithms for the fractional calculus: A selection of numerical methods*,Comp. Meth. Appl. Mech. Eng. 194, 743-773.

[2] R. Pavlović, P. Kozić, P.M. Rajković, *Influence of transverse shear on the stochastic instability of viscoelastic beam*, International Journal of Solids and Structures, Vol. 38, Issues 38-39 (2001) 6829-6837.

Buckling and postbuckling analysis of nanotube

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Due to the development of technology the analysis of nanotubes has become a very interested field of research. One important direction of this research is the problem of analyzing the stability boundary of an elastic nanotube conveying fluid. Namely, because of excellent mechanical properties and perfect hollow geometry, the carbon nanotube (CNT) promises many new applications in nanobiological devices and nanomechanical systems such as fluid storage, fluid transport, and drug delivery. Instead of analyzing a nanotube conveying fluid we investigate, mathematically equivalent, the problem of an inextensible string that is pulled through a nanotube.

The usual way of investigation of mechanical behavior of nanotubes is the use of molecular dynamics simulations or continuum mechanics. In this paper we use the methods of continuum mechanics.

Thus, we consider a nanotube, pinned at both ends, through which a string is pulled with constant velocity. Inside the nanotube there is a friction force between the nanotube and the string. The constitutive equation for the nanotube is taken in the form of non-local constitutive relation, suggested by Eringen in 1983. Since we study the divergence type of instability only nonlinear equilibrium equations for the nanotube are derived. By using the Liapunov-Schmidt method the analysis of the lowest bifurcation point of these equations is performed. The obtained results are:

1. The system of non-linear differential equations describing the equilibrium configuration of the nanotube can be reduced to two non-linear differential equations.
2. The non-local Eringen theory and the classical Timoshenko beam theory lead to the same type of non-linear governing equations.
3. The explicit form of stability boundary (critical velocity) is obtained. Also, the small length scale parameter, representing the influence of nonlocal effects, decreases the stability boundary.
4. The bifurcation pattern at the critical velocity is super-critical for all values of small length scale parameter. This means that the non-local effect does not influence the bifurcation pattern.

Asymptotic solutions of quasilinear equations and the problem of instability of equlibria of mechanical systems

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In the first part of the lecture we shall consider asymptotic solutions of quasilinear equations. Consider the system of ordinary differential equations of the form

$$\frac{d\mathbf{z}}{d\tau} = U\mathbf{z} + \mathbf{\Psi}(\tau, \mathbf{z}), \quad \mathbf{z} \in R^s, \tag{1}$$

where U is constant $s \times s$ matrix, $\mathbf{\Psi}(\tau, \mathbf{z})$ a continuous $s-$vector for which $\lim_{\tau\to\infty} \mathbf{\Psi}(\tau, \mathbf{z}) = \mathbf{0}$ holds. By using the linear transformation, matrix U can be expressed in the form $U = \mathrm{diag}(N, P)$, where $\mathrm{Re}(\lambda_i(N)) < 0$, $i = 1, \ldots, k$, and $\mathrm{Re}(\lambda_j(P)) \geq 0$, $j = k+1, \ldots, s$.

Green function corresponding to the operator U takes the form

$$\Gamma(U) = \begin{cases} (\exp U\zeta)\mathrm{diag}(I_k, 0), & \zeta > 0 \\ -(\exp U\zeta)\mathrm{diag}(0, I_{s-k}), & \zeta < 0 \end{cases}$$

in which I_k and I_{s-k} denote $s \times s$ and $(s-k) \times (s-k)$ unit matrices, respectively. Green function Γ has the following properties:

(i) for each $\zeta \neq 0$ the function $\Gamma(\zeta)$ is continuously differentiable and satisfies the equation $d\Gamma/d\zeta = U \cdot \Gamma(\zeta)$;

(ii) for $\zeta = 0$, $\Gamma(0^+) - \Gamma(0^-) = I$, where I is the $s \times s$ unit matrix;

(iii) for every constant $\gamma > 0$ and every constant $\alpha > 0$ which satisfy $\alpha + \gamma < \min(\mathrm{Re}(\lambda_i(-N)))$, there is a sufficiently large positive constant K for which $\|\Gamma(\zeta)\| \leq K \exp(-(\alpha+\gamma)\zeta)$, $\zeta > 0$, and $\|\Gamma(\zeta)\| \leq K \exp(-\gamma\zeta)$, $\zeta < 0$, respectively.

The following theorem determines the asymptotic behavior of the solution $\mathbf{z}(\tau)$.

Theorem 1. Let on the set $H = \{(\tau, \mathbf{z}) : \tau \geq \tau_0, \|\mathbf{z}\| \leq \exp(-(\alpha/2+\gamma)\tau)\}$, $\tau_0 > 0$, the following conditions be satisfied: (a) $\mathbf{\Psi}(\tau, \mathbf{z})$ is continuous function; (b) for $(\tau, \mathbf{z})$, $(\tau, \mathbf{z}^{(1)})$, $(\tau, \mathbf{z}^{(2)}) \in H$ there holds $\|\mathbf{\Psi}(\tau, \mathbf{z})\| \leq M \exp(-(\alpha + 2\gamma)\tau)$ and

$$\|\mathbf{\Psi}(\tau, \mathbf{z}^{(1)}) - \mathbf{\Psi}(\tau, \mathbf{z}^{(2)})\| \leq M \exp(-(\alpha/2 + \gamma)\tau)\|\mathbf{z}^{(1)}\mathbf{z}^{(2)}\|, \quad M > 0;$$

(c) there is at least one eigenvalue of the matrix U with negative real part.

Then there exists $a > \tau_0$ and continuously differentiable solution $\mathbf{z} : (a, \infty) \to R^s$ of Eq. (1) which satisfies $\|\mathbf{z}\| = O(\exp(-(\alpha/2 + \gamma)\tau))$.

Second part of the lecture is devoted to the study of instability of equilibrium of non-holonomic system. Consider a scleronomic mechanical system with generalized coordinates $\mathbf{x} = (x_1, \ldots, x_n)^T$ (where T denotes the transpose) subject to constraints linear in the velocities:

$$\mathbf{B}(\mathbf{x})\dot{\mathbf{x}} = \mathbf{0} \tag{2}$$

where $\mathbf{B}(\mathbf{x})$ is $n \times m$ matrix of rank m. Let $T = (1/2)\dot{\mathbf{x}}^T\mathbf{A}(\mathbf{x})\dot{\mathbf{x}}$ be the kinetic energy ($\mathbf{A}(\mathbf{x})$ is symmetric positive definite $n \times n$ matrix for each $\mathbf{x} \in R^n$) and Π the potential of the force field.

Let the system have an equilibrium position $\mathbf{x} = \mathbf{0}$. We shall introduce the following hypothesis (H):

(1) $\mathbf{A}(\mathbf{x})$ and $\mathbf{B}(\mathbf{x})$ are $C^2(R^n)$;

(2) $\Pi(\mathbf{x})$, $\Pi_p(\mathbf{x}) : R^n - \{0\} \to R^n$ are $C^2(R^n)$ and $C^3(R^n)$, respectively, $\Pi_p(\mathbf{x})$ being homogeneous function of degree $p > 1$;

(3) for $i = 0, 1, 2$, $\Pi^{(i)}(\mathbf{x}) = \Pi_p^{(i)}(\mathbf{x}) + O\left(\|\mathbf{x}\|^{p+\varepsilon-i}\right)$, when $\mathbf{x} \to \mathbf{0}$, $0 < \varepsilon \leq 1$.

Let π be the $(n-m)-$dimensional subspace determined in the following way: $\pi = \{\mathbf{x} \in R^n : \mathbf{B}_0^T\mathbf{x} = \mathbf{0}\}$, $\mathbf{B}_0 = \mathbf{B}(\mathbf{0})$. Let $\hat{\Pi}_p$ be restriction of Π_p to π, and thus homogeneous function of degree p.

Theorem 2. The equilibrium state $\mathbf{x} = \mathbf{0}$, $\dot{\mathbf{x}} = \mathbf{0}$ is unstable if the following conditions are fulfilled:

(a) the potential $\Pi(\mathbf{x})$ and matrices $\mathbf{A}(\mathbf{x})$ and $\mathbf{B}(\mathbf{x})$ satisfy the hypothesis (H);

(b) the function $\hat{\Pi}_p$ has no minimum at $\mathbf{x} = \mathbf{0}$.

Theorem 2 generalizes the result of the paper [1]. We must point out, that Theorem 2 holds also in the case of integrable constraints (2). Proof of the Theorem 2 is be based on the fact that equations of motion of the described mechanical system, although strictly nonlinear, can be brought to the form of the equations (1), see [2].

References

[1] V. V. Kozlov, Soviet. Math. Dokl. 33 (3) 654-656 (In Russian) (1986)

[2] M. Vesković, V. Čović, Mathematical and Computer Modeling 45 (9-10): 1145-1156 (2007)

Weyl asymptotics and Dirichlet divisors

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We consider tensor products of self-adjoint partial differential operators or pseudo-differential operators on compact manifolds and Euclidean spaces. The Weyl asymptotics for the counting functions are then determined by solving related problems for Dirichlet divisors (results in collaboration with T.Gramchev, S.Pilipovic, J.Vindas).

Extended thermodynamics of real gases

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The kinetic theory and the Extended thermodynamics (ET) are important theories for rarefied non-equilibrium gas. Nevertheless the weak point is that the range is limited to monatomic gas. In this talk we want to present recent new approach to deduce hyperbolic system for real gas not necessary rarefied. In the first part of the talk we study extended thermodynamics of dense gases by adopting the system of field equations with a different hierarchy structure to that adopted in the previous works.

It is the theory of 14 fields of mass density, velocity, temperature, viscous stress, dynamic pressure and heat flux. As a result, all the constitutive equations can be determined explicitly by the caloric and thermal equations of state as in the case of monoatomic gas.

It is shown that the rarefied-gas limit of the theory is consistent with the kinetic theory of gases. In the second part we specialized the result to the physical interesting case of rarefied polyatomic gas and we show a perfect coincidence between ET and the procedure based upon maximum entropy principle. The main difference with respect to usual procedure is existence of two hierarchies of macroscopic equations for moments of suitable distribution function, in which the internal energy of a molecule is taken into account.

References

[1] I. Müller and T. Ruggeri (1998) *Rational Extended Thermodynamics*, 2nd ed., Springer Tracts in Natural Philosophy **37**, Springer-Verlag, New York.

[2] G. Boillat and T. Ruggeri (1997) Moment equations in the kinetic theory of gases and wave velocities, *Continuum Mech. Thermodyn.*, **9**, 205–212.

[3] T. Arima, S. Taniguchi, T. Ruggeri and M. Sugiyama (2011) Extended thermodynamics of dense gases, *Continuum Mech. Thermodyn.*, doi: 10.1007/s00161-011-0213-x.

[4] M. Pavić, T. Ruggeri and S. Simić, Maximum entropy principle for rarefied polyatomic gases. Submitted to Physica A (2012).

Anisotropic elasticity damage tensor in continuum damage mechanics

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The problem of estimating the effective elastic properties of a damage solid has been intensively studied in the last four decades.

In continuum damage mechanics, usually a phenomenological approach is adopted. In this approach, the most important concept is that of the Representative Volume Element (RVE). The discontinuous and discrete elements of damage are not considered within the RVE; rather their combined effects are lumped together through the use of a macroscopic internal variable. In this way, the formulation may be derived consistently using sound mechanical and thermodynamic principles.

The formulation is presented within the framework of the usual classical theory of elasticity.

At any given state of damage, the elastic portion of the material response will be characterized by a fourth-order tensor $\mathbb{C}$ of the damaged elastic moduli just as the fourth order tensor $\mathbf{E}$ describes the elastic response of the virgin material. In general, one may expect that the damage moduli depend on both the undamage values and on some measure of the damage level, i.e. $\mathbb{C}$ ($\mathbf{E}$, damage level).

In most of the existing damage theories, the damaged elastic strain-stress (or stress-strain) response is formulated by using the notion of effective stress (strain) and the hypothesis of strain (stress) equivalence or stress-energy (strain-energy) equivalence (Lemaitre and Chaboche, 1985; Cauvin and Testa, 1999).

The damage variable (or tensor), based on the effective stress concept, represents average material degradation which reflects the various types of damage at the micro-scale level like nucleation and growth of voids, cracks, cavities, micro-cracks, and other microscopic defects.

In the present paper it is shown that closed form solution is possible in the process of deriving damage tensor components, starting from the principle of strain energy equivalence. It is shown that damaged tensors are of the fourth order, and they are derived for orthotropic, hexagonal, cubic and isotropic damage. To the best knowledge of the authors nobody derived damage tensor in closed form solution up to now.

References

[1] Lemaitre J., Chaboche, J.-L., "Mechanics of Solid Materials", Cambridge University Press, (1990).

[2] G.Z.Voyiadjis and P.I. Kattan, "Damage Mechanics with Fabric Tensors", Mechanics of Advanced Materials and Structures, 13, 285-301, 2006.

[3] Cauvin, A., and Testa, R., (1999) Damage mechanics: Basic variables in continuum theories, *International Journal of Solids and Structures*, **36**, pp. 747-761.

[4] Gurtin, M.E. *The linear theory of elasticity*, Handbuch der Physik, $\mathbf{6a}_2$, Berlin, Springer-Verlag (1972).

[5] Jarić, J., Kuzmanović, D., and Golubović, Z., "On Tensors of Elasticity", Theoret. Appl. Mech. Vol. 35, No. 1-3, pp. 119-136, 2008.

[6] Srinivasan, T.P., & Nigam, S.D., *Invariant Elastic Constants for Crysrals*, J. Math. Mech., Vol. 19., No. 5, (1969).

[7] J. Jarić, P. Cvetković, Z. Golubović, D. Kuzmanović, "Advances in Continuum mechanics", Monographical booklets in applied & computer mathematics, MB-25/PAMM, Budapest 2002.

[8] Lubarda, V.A., and Krajčinović, D., *Damage tensors and the crack density distribution*, Int. J. Enging. Sci., No. 20, pp. 2859-2877, (1993).

Some applications of fractional calculus on control problems in robotics and system stability

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In recent years, there have been extensive research activities related to applications of fractional calculus (FC), [5] in nonlinear dynamics, mechatronics as well as control theory. In this paper, they are presented recently obtained results which are related to applications of fractional calculus in mechanics - specially stability and control issues. Some of these results [1-4] are presented at the *Fifth symposium of fractional differentiation and its applications FDA2012*, was held at the Hohai University, Nanjing, China in the period of May 14-May 17, 2012. Also, fractional order dynamic systems and controllers have been increasing in interest in many areas of science and engineering in the last few years.In that way, our objective of using fractional calculus is to apply the fractional order controller to enhance the system control performance as well as it has better disturbance rejection ratios and less sensitivity to plant parameter variations.

First, they are introduced and obtained the new algorithms of fractional order PID control based on genetic algorithms in the position control of a 3 DOF's robotic system driven by DC motors. Then, the main task is to find out the optimal settings for a fractional $PI^{\alpha}D^{\beta}$ controller in order to fulfill the proposed design specifications for the closed-loop system. In addition, this method allows the optimal design of all major parameters of a fractional PID controller and then enhances the flexibility and capability of the PID controller. Last,in simulations, they are compared step responses of these two optimal controllers where it will be shown that fractional order PID controller improves transient response as well as provides more robustness in than conventional PID.

Second, we propose sufficient conditions for finite time stability for the (non)homogeneous fractional order systems with time delay. Specially, the problem of finite time stability with respect to some of the variables (partial stability) is considered. Namely, along with the formulation of the problem of stability to all variables, Lyapunov also formulated a more general problem on the stability to a given part of variables (but not all variables) determining the state of a system,[6].The problem of the stability of motion with respect to some of the variables also known as partial stability arises naturally in applications. So, in this presentation, it will be proposed finite time partial stability test procedure of perturbed (non) linear (non)autonomous time varying delay fractional order systems. Time-delay is assumed to be varying with time but its upper bound is assumed to be known over given time interval. New stability criteria for this class of fractional order systems will be derived using “classical” Bellman-Gronwall inequality,as well as another another suitable inequality, [7]. Last,a numerical example is provided to illustrate the application of the proposed stability procedure.

Third, some attention is devoted to the problem of stability of linear discrete-time fractional order systems is addressed, [8]. It was shown that some stability criteria for discrete time-delay systems could be applied with small changes to discrete fractional order state-space systems. Accordingly, simple conditions for the stability and robust stability of a particular class of linear discrete time-delay systems are derived. These results are modified and used for checking the stability of discrete-time fractional order systems. The systems under consideration involve time delays in the state and parameter uncertainties. The parameter uncertainties are assumed to be time-varying and norm bounded.

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References

[1] M.Lazarević, Finite Time Partial Stability of Fractional Order Time Delay Systems, No147, (2012),FDA2012,Nanjing,May,China. (award for the Best poster).

[2] S.A. Batalov, M.P. Lazareviæ,A. Hace, K.Jezernik, A Chattering-free Fractional PDa Sliding-mode Control of a 3-DOF Robot System Driven by DC motors, No130,(2012) FDA2012, Nanjing, May,China.

[3] Lj. Bučanović, M. Lazarević,S. Batalov, Optimal Fractional Order PID Control Of Expansion Turbine In The Air Production Cryogenic Liquid,No128, (2012),FDA2012, Nanjing, May, China. (award for the Best oral presentation)

[4] M.Lazarević, Iterative Learning Control for Fractional Order Time Delay System - PDa Type,No 259,(2012),FDA2012, Nanjing,May,China.

[5] Sabatier J., O. P. Agrawal, J.A. Tenreiro Machado:Advances in Fractional Calculus: Theoretical Developments and Appl. in Physics and Engineering, Springer,(2007),p.55.

[6] Vorotnikov, V., Partial Stability and Control: The State-of-the-art and Development Prospects, Automation and Remote Control, Vol. 66, No. 4, (2005).pp. 511-561.

[7] Lazarević, M., A. Spasić, Finite-time stability analysis of fractional order time delay systems: Gronwall‘s Approach, Mathematical and Computer Modelling,(2009),49, pp.475-481.

[8] Stojanović. S, Debeljković D., M Lazarević, Stability of Discrete-Time Fractional Order Systems: An Approach Based on Stability of Discrete-Time Integer Order Time-Delay Systems, chapter in a book Advanced Topics on Applications of Fractional Calculus on Cont.rol Prob.,WSEAS (accepted),2012.pp.59-78.

Microbearing gas flow modeling by fractional derivative for entire Knudsen number range

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Rarefied gas flow is encountered in many technical applications as well as in scientific inquiry. It may appear in low-pressure or vacuum environmental conditions and, on the other hand at standard atmospheric conditions. First category include gas flow in the devices used in hypersonic space vehicles and in several types of vacuum instruments, while the second category relates to the gas flow in micro/nano-electro-mechanical-systems (MEMS, NEMS) with characteristic dimension of the order of and . In these systems the ratio between the mean free path of the molecules and the characteristic length, which is defined as the Knudsen number (Kn), is not negligible and continuum approach breaks down. As a consequence gas slips along the wall and classical no-slip boundary conditions are no more valid. In the range $10^{-2} < Kn < 10^{-1}$, known as the slip flow regime, gas flow still obeys continuum i.e. Navier-Stokes equations, but now with slip boundary conditions at the walls. In the range $10^{-1} < Kn < 10$ (transitional flow regime) more complex Burnett equations have to be applied. The accuracy of the Burnett equations is of the order $O(Kn^2)$ and they are solved under the boundary conditions of the same second order accuracy. Besides, the individual particle-based direct simulation Monte Carlo (DSMC) approach might be employed. Finally, for $Kn > 10$ the gas flow is considered as a free molecular flow amenable to the methods of kinetic theory of gases.

In this paper rarefied compressible two-dimensional gas flows in microbearings that are often part of MEMS and NEMS are treated. Instead of different approaches for slip velocity for the three rarefied gas flow regimes, slip at the boundaries is modeled by fractional derivative for the whole Knudsen number range. For this purpose a version of Caputo derivative is defined, with the order α defined as a function of the local value of the Knudsen number in the microbearing. For no-slip boundary conditions i.e. for continuum flow regime $\alpha = 0$, while for free molecular flow when the Knudsen number approaches infinity $\alpha \to 1$. The correlation between α and Kn is derived in the following way. The flow rate coefficient of Poiseuille flow Q_P is calculated for various Knudsen numbers by utilizing the numerical solution of the Boltzmann equation obtained by Fukui and Kaneko (1988). The obtained values of Q_P for specified Kn numbers are used for the derivation of the analytical relation between α and kn. Such a universal boundary condition that defines velocity at the wall for an arbitrary Knudsen number value is incorporated in the system of continuity and momentum equation, which leads to the general slip-corrected Reynolds lubrication equation. It is shown that it possesses the analytical solution which is obtained by a suitable transformation of the independent variable (Stevanovic and Djordjevic, 2012). It provides the mass flow rate as well as the pressure distribution in the microbearing for a specified bearing number Λ, the reference Knudsen number at the exit cross section $Kn_e$and the ratio of the inlet and exit microbearing height.

The results for a wide range of the Knudsen number and the continuum flow conditions, obtained by the general analytical solution from this paper, are in excellent agreement with Fukui and Kaneko's (1988) numerical solution of the Boltzmann equation.

References

[1] S. Fukui, R. Kaneko, Analysis of ultra-thin gas film lubrication based on linearized Boltzmann equation. First report-derivation of a generalized lubrication equation including thermal creep flow, J. Tribol. 110 (1988), 253-262.

[2] N. D. Stevanovic , V. D. Djordjevic, The exact analytical solution for the gas lubricated bearing in the slip and continuum flow regime, Publications de l'Institute mathematique 91 (2012), 83-93.

Diffusion asymptotics of a kinetic model for gaseous mixtures

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In this work, we consider the non-reactive fully elastic Boltzmann equations for mixtures constituted with $I \geq 2$ species. Each species $\mathcal{A}_i$ of the mixture, $1 \leq i \leq I$, is described by a microscopic density function f_i, which depends on time $t \in \mathbb{R}_+$, space position $x \in \mathbb{R}^3$ and molecular velocity $v \in \mathbb{R}^3$, and is nonnegative. More precisely, $f_i(t, x, v)\,\mathrm{d}x\mathrm{d}v$ allows to quantify the number of molecules of species $\mathcal{A}_i$ at time t in an elementary volume of size $\mathrm{d}x$, and whose velocities equal v up to $\mathrm{d}v$. We can also define the macroscopic density n_i of each species $\mathcal{A}_i$ by

$$n_i(t, x) = \int_{\mathbb{R}^3} f_i(t, x, v)\,\mathrm{d}v.$$

We focus on the diffusive limit of the Boltzmann equations obtained from the framework of the classical diffusive scaling, where the scaling parameter is the mean free path. We look for each distribution function f_i, $1 \leq i \leq I$, as a formal power series in scaling parameter, replace it into i-th Boltzmann equation and identify the same order terms.

The order -1 allows to find the zero-th order term of the series: Maxwell functions described in the H-theorem. Therefore, each distribution function f_i, $1 \leq i \leq I$, can be seen as a perturbation of the equilibrium:

$$f_i(t, x, v) = M_i(v)\, n_i(t, x) + \varepsilon\, M_i(v)^{1/2}\, g_i(t, x, v) + ..., \quad \forall t \geq 0,\ \forall x, v \in \mathbb{R}^3,$$

where $M_i(v)$ is the normalized, centred Maxwell function $M_i(v) = \left(\frac{m_i}{2\pi}\right)^{3/2} e^{-\frac{m_i}{2}v^2}$, $\forall v \in \mathbb{R}^3$.

The zero-th order leads to a linear functional equation in the velocity variable:

$$(\mathcal{K} - \nu\,\mathrm{Id})\, g = \left(M_i^{1/2}\,(v \cdot \nabla_x n_i)\right)_{1 \leq i \leq I},$$

where $\nu = \nu(v)$ is positive function. The main result of this work is a theorem which ensures existence of the solution $g(t, x, \cdot) \in L^2(\mathbb{R}^3_v)^I$ under two assumptions. First assumption refers to cross section included into collisional operators and consists on a general condition satisfied by hard spheres and all cutoff power-law potentials. Second assumption is a requirement that total number density $\sum_{i=1}^{I} n_i(t, x)$ does not depend on space position x. It could be fulfilled, for example, in the equimolar diffusion situation (common in closed experimental settings) if we suppose that the initial value of total number density does not depend on x. Proof of the theorem is based on application of the Fredholm alternative provided compactness of the operator $\mathcal{K}$ from $L^2(\mathbb{R}^3_v)^I$ to $L^2(\mathbb{R}^3_v)^I$ is known. The proof of compactness brings out new method which we propose for treating terms involving particles with different masses. Namely, techniques introduced by Grad, who studied the formal small free path limit for the monatomic and monospecies Boltzmann equation, can be partly extended to the case of multispecies mixtures i.e. only when considering collisions between same mass particles. The reason lies in a symmetry property of the collision process when colliding molecules are of same masses. This property is completely lost when masses of molecules are different. We hence propose a new approach to the problem which, in turn, only works when collisions involve particles with different masses.

The first order allows to obtain the usual continuity equation $\partial_t n_i + \nabla_x \cdot N_i = 0$, where the flux N_i of species $\mathcal{A}_i$ is given by

$$N_i(t,x) = \int_{\mathbb{R}^3} v\, g_i(t,x,v)\, M_i(v)^{1/2}\, \mathrm{d}v, \qquad \forall x, v \qquad 1 \le i \le I.$$

Since g_i only depends on (n_j), the flux N_i shares the same property. Consequently, the continuity equations give a system, which only involves (n_j). This concludes the work.

References

[1] C. Bardos, F. Golse, and C. Levermore. Fluid dynamic limits of kinetic equations. I. Formal derivations. J. Statist. Phys., 63(1-2):323344, 1991.

[2] L. Boudin, B. Grec, and F. Salvarani. The Maxwell-Stefan diffusion limit for a kinetic model of mixtures. Submitted.

[3] C. Cercignani. The Boltzmann equation and its applications, volume 67 of Applied Mathematical Sciences. Springer-Verlag, New York, 1988.

[4] L. Desvillettes, R. Monaco, and F. Salvarani. A kinetic model for chemically reacting gases. In Modelling and numerics of kinetic dissipative systems, pages 119132. Nova Sci. Publ., Hauppauge, NY, 2006.

[5] H. Grad. Asymptotic theory of the Boltzmann equation. II. In Rarefied Gas Dynamics (Proc. 3rd Internat. Sympos., Palais de l'UNESCO, Paris, 1962), Vol. I, pages 2659. Academic Press, New York, 1963.

[6] L. Sirovich. Kinetic modeling of gas mixtures. Phys. Fluids, 5:908918, 1962.

Shock structure and temperature overshoot in multi-temperature model of binary mixture

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This lecture addresses the problem of shock structure in macroscopic multi-temperature model of gaseous mixtures, recently established within the framework of extended thermodynamics [1]. Basic assumption is that each component of the mixture is ideal gas described by its own fields of density, velocity and temperature $(\rho_\alpha, \mathbf{v}_\alpha, T_\alpha)$. As a consequence, governing equations are balance laws of mass, momentum and energy of each component, where source terms take into account mutual interaction of the components.

$$\frac{\partial \rho_\alpha}{\partial t} + \operatorname{div}(\rho_\alpha \mathbf{v}_\alpha) = \tau_\alpha,$$
$$\frac{\partial (\rho_\alpha \mathbf{v}_\alpha)}{\partial t} + \operatorname{div}(\rho_\alpha \mathbf{v}_\alpha \otimes \mathbf{v}_\alpha - \mathbf{t}_\alpha) = \mathbf{m}_\alpha,$$
$$\frac{\partial \left(\frac{1}{2}\rho_\alpha v_\alpha^2 + \rho_\alpha \varepsilon_\alpha\right)}{\partial t} + \operatorname{div}\left\{\left(\frac{1}{2}\rho_\alpha v_\alpha^2 + \rho_\alpha \varepsilon_\alpha\right)\mathbf{v}_\alpha - \mathbf{t}_\alpha \mathbf{v}_\alpha + \mathbf{q}_\alpha\right\} = e_\alpha,$$

The study is restricted to the simplest case of binary mixture of non-viscous and non-heat-conducting inert gases. The simplicity of the model that allows to test the hypothesis that mass difference between the constituents is the main driving agent which tears their temperatures apart.

By assuming the traveling wave solution we studied the shock structure and eventually confirmed the aforementioned hypothesis. We also studied the temperature overshoot of the heavier component of binary mixture. In particular, its dependence on mass ratio (as a parameter) and concentration and Mach number in unperturbed state is analyzed.

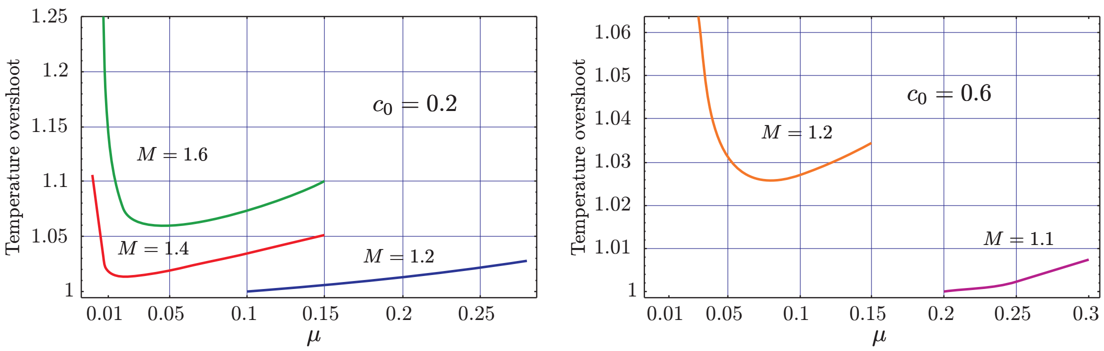

Figure 1. Temperature overshoot estimation for different values of mass ratio μ and Mach number M

Temperature overshoot is analyzed for two different concentrations, $c_0 = 0.2$ and $c_0 = 0.6$, and different values of Mach number. It was observed that for given concentration and Mach number there are two possible bounds for the values of mass ratio: lower one (if exists) is due to vanishing of temperature overshoot (cases $c_0 = 0.2$, $M = 1.2$ and $c_0 = 0.6$, $M = 1.1$), while higher one indicates appearance of sub-shock, i.e. discontinuous shock structure. It is to be expected that temperature overshoot grows with the increase of Mach number and the decrease of mass ratio. However, our numerical study showed that dependence of the temperature overshoot on mass ratio is non-monotonous. There exists a critical value of mass ratio μ_{crit} for which temperature overshoot attains the minimum. For $\mu < \mu_{crit}$ temperature overshoot decreases with the increase of mass ratio. For $\mu > \mu_{crit}$ temperature overshoot increases with the increase of mass ratio.

This is the first step in systematic study of temperature overshoot in binary mixtures. At this moment we cannot give precise physical explanation of this phenomenon, especially when we have in mind that diffusion is the only dissipative mechanism taken into account. It remains to be studied what is the influence of viscosity and heat conduction on temperature overshoot.

References

[1] T. Ruggeri and S. Simić, On the hyperbolic system of a mixture of eulerian fluids: a comparison between single- and multi-temperature models, *Mathematical Methods in the Applied Sciences* 30 (2007), No. 7, 827–849.

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